UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
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Complex Analysis II

(Funktionentheorie II)

Introduction to Geometric Function Theory and Schramm-Loewner Evolution

Lecture Winter Semester 2017/2018

In geometric function theory, holomorphic functions are regarded as mappings between subsets of the complex plane and one aims at understanding the rich interplay between their geometric and analytic properties. Of interest are especially *conformal maps*, namely holomorphic functions that are additionally injective.
Among the numerous powerful tools that were developed for this purpose is the so-called *Loewner evolution*. It can be seen as a dynamical process $(g_t)_{t\geq 0}$ on the set of conformal maps, which describes the evolution of some curve in a fixed simply connected domain. On the complex upper half plane $\mathbb{H}$, it is determined by the *chordal Loewner equation*

$$\frac{\partial}{\partial t} g_t(z) = \frac{2}{g_t(z) - \lambda(t)}, \qquad g_0(z) = z$$

with respect to some *driving function* $\lambda : [0, \infty) \to \mathbb{R}$.
In 2000, Oded Schramm added a stochastic component to that theory by considering driving functions λ of the form $\lambda(t) = \sqrt{\kappa} B_t$ for $\kappa > 0$ and with $(B_t)_{t\geq 0}$ being the one-dimensional Brownian motion. This results in a kind of Brownian motion on the space of conformal maps, called the *stochastic Loewner evolution* SLE_κ, i.e., a family $(g_t)_{t\geq 0}$ of random conformal maps associated to some random curve in $\mathbb{H}$.
The character of SLE_κ strongly depends on κ. Their relevance lies in the fact that they are (conjectured to be) the *scaling limit* of various discrete random processes appearing in mathematics and physics, such as the loop-erased random walk and percolation.
With Wendelin Werner (2006) and Stanislav Smirnov (2010), two mathematicians have been awarded the Fields medal for their contributions to this fascinating field.

In this lecture, which is formally a continuation of the lecture *Funktionentheorie*, we give an introduction to the basics of both geometric function theory and stochastic Loewner evolution. We will also provide the needed facts from probability theory and stochastic calculus. Participants are supposed to have some prior knowledge on basic measure theory and complex analysis (such as *Analysis III* and *Funktionentheorie*), but no prerequisites on probability theory are assumed.

Time and Place: **Tuesday, 12 – 14, Seminar Room 10, Building E2 4**

For further information, please contact Tobias Mai (`mai@math.uni-sb.de`, room 225 in building E2 4). See also:

`https://www.math.uni-sb.de/ag/speicher/lehre.html`