UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
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Assignments for the lecture *Complex Analysis II*
Winter term 2017/2018

Assignment 2B
for the tutorial on *Tuesday, November 21, 4:15 pm* (in Seminar Room 10)

Problem 1. Let $f : \mathbb{D} \to \mathbb{C}$ be a schlicht holomorphic function. We define

$$d_f(z) := \operatorname{dist}(f(z), \partial f(\mathbb{D})) := \inf_{w \in \partial f(\mathbb{D})} |f(z) - w| \qquad \text{for } z \in \mathbb{D}.$$

Prove that for all $z \in \mathbb{D}$

$$\frac{1}{4}(1 - |z|^2)|f'(z)| \leq d_f(z) \leq (1 - |z|^2)|f'(z)|.$$

What does this give in the special case $f \in \mathcal{S}$ and $z = 0$?

Hint: For any given $a \in \mathbb{D}$, consider the Koebe transform of f associated to

$$\phi : \ \mathbb{D} \to \mathbb{D}, \quad z \mapsto \frac{z + a}{1 + \overline{a}z}$$

Problem 2. Consider an open set $\Omega \subseteq \mathbb{C}$ that is *symmetric with respect to the real axis* (i.e., it holds true that $\overline{z} \in \Omega$ for all $z \in \Omega$). We put

$$\Omega_+ := \{z \in \Omega \mid \operatorname{Im}(z) > 0\} \qquad \text{and} \qquad \Omega_- := \{z \in \Omega \mid \operatorname{Im}(z) < 0\}$$

and in addition $L := \Omega \cap \mathbb{R}$.

Prove the *Schwarz reflection principle*: If f is a holomorphic function on Ω_+ that admits a continuous extension to $\Omega_+ \cup L$ satisfying $f(L) \subseteq \mathbb{R}$, then

$$\tilde{f} : \ \Omega \to \mathbb{C}, \ z \mapsto \begin{cases} f(z) & \text{for } z \in \Omega_+ \cup L \\ \overline{f(\overline{z})} & \text{for } z \in \Omega_- \end{cases}$$

defines a holomorphic extension of f to Ω.