UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
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Assignments for the lecture *Complex Analysis II*
Winter term 2017/2018

Assignment 3A
for the tutorial on *Tuesday, December 5, 4:15 pm* (in Seminar Room 10)

Problem 1. Prove Lemma 3.3 of the lecture:

> *Let $(f_n)_{n=1}^\infty$ be a sequence of schlicht holomorphic functions $f_n : \mathbb{D} \to \mathbb{C}$ that converges compactly (i.e., uniformly on compact subsets of $\mathbb{D}$) to some schlicht holomorphic function $f : \mathbb{D} \to \mathbb{C}$. Then, for any given compact set*
>
> $$K \subset f(\mathbb{D}),$$
>
> *there is some $N \in \mathbb{N}$, such that*
>
> $$K \subset f_n(\mathbb{D}) \qquad \text{for all } n \geq N.$$

Hint: Use Rouché's theorem.

Problem 2. Prove part (ii) of Theorem 3.4 of the lecture:

> *Consider the situation $(*)$ that was described in Chapter 3, i.e., suppose that*
>
> - *$(G_n)_{n=1}^\infty$ is a sequence of simply connected domains $G_n \subsetneq \mathbb{C}$ with $0 \in G_n$;*
> - *$(f_n)_{n=1}^\infty$ is the corresponding sequence of biholomorphic maps $f_n : \mathbb{D} \to G_n$ satisfying $f_n(0) = 0$ and $f_n'(0) > 0$.*
>
> *Then $(f_n)_{n=1}^\infty$ converges compactly on $\mathbb{D}$ to 0, if and only if no subsequence of $(G_n)_{n=1}^\infty$ has a kernel.*

Hint: Use the Schwarz lemma and Theorem 2.8.