UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
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Assignments for the lecture *Complex Analysis II*
Winter term 2017/2018

Assignment 5
for the tutorial on *Tuesday, **January 16, 2018 (!)**, 4:15 pm* (in Seminar Room 10)

It is fundamental for the understanding of Brownian motion to be familiar with the nature of (multivariate) Gaussian distributions, in particular of their behaviour under linear transformations. Therefore you should look it up, e.g. in the library or on the Web.

Problem 1. Let X_1 and X_2 be two Gaussian random variables with $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$. Assume that X_1 and X_2 are independent.

1. Show that $X_1 + X_2 \sim \mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$.
2. Let $a, b \in \mathbb{R}_+$, and for X_1 and X_2 the same assumptions as above. What is then $aX_1 + bX_2$?

Problem 2. Invariances of Brownian motion. Let $(B_t)_{r\in\mathbb{R}_+}$ be a standard BM. Show the following properties

1. $(-B_t)_{t\in\mathbb{R}_+}$ is a BM.
2. For $0 < c$, the rescaled Brownian motion $(\frac{1}{c}B_{c^2t})_{t\in\mathbb{R}_+}$ is a BM.

Problem 3. (Bessel process) Let B_t be an n-dim. BM with stochastic Euclidean norm $R_t(\omega) := \|B_t(\omega)\|$. Consider the function $r : \mathbb{R}^n \to \mathbb{R}$, $x \mapsto \|x\|$. Use the Ito formula in order to determine what dR_t is.

Problem 4.
1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and $\mathcal{G} \subset \mathcal{F}$ a sub-σ-algebra. Assume that the random variable X is independent of $\mathcal{G}$. Show that $\mathbb{E}[X|\mathcal{G}] = \mathbb{E}[X]$.
2. Let $f : [a, b] \to \mathbb{R}$ be a monotonic function. What is its variation on the interval $[a, b]$?