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Exercises for the lecture ***Operator algebras (Functional analysis II)***
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Sheet 7
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Let M be a von Neumann algebra , $e \in M$ a projection. We say that

(i) $e \neq 0$ is *minimal*, if for every projection $f \in M$ with $f \leq e$ we have $f = 0$ or $f = e$.

(ii) e is *finite* if $e \sim f \leq e$ implies $e = f$.

Exercise 1 (10 points)**.** Consider the von Neumann algebra $M = B(H)$, where H is a separable Hilbert space and let $e, f \in B(H)$ be projections. Show that

(a) $e \sim f$ if and only if $\dim eH = \dim fH$.

(b) $e \precsim f$ if and only if $\dim eH \leq \dim fH$.

(c) e is minimal if and only if e is a projection of rank 1.

(d) e is finite if and only if eH is finite dimensional.

Exercise 2 (10 points)**.** Let M be a von Neumann algebra such that 1 is a finite projection. Prove the following statements:

(a) Every projection $e \in M$ is finite.

(b) If $e \sim f$ then $1 - e \sim 1 - f$ for projections $e, f \in M$.

(c) If $e \sim f$ then there exists a unitary $u \in M$ such that $f = ueu^*$.

(d) The statements (b) and (c) are false if M is a von Neumann algebra, where 1 is not finite.

Hint: Consider the unilateral Shift.