



# Von Neumann algebras, subfactors, and planar algebras

Weiterführende Vorlesung im Sommersemester 2016

Operator algebras are generalizations of matrix algebras to the infinite dimensional setting; their theory, however, becomes much more involved and combines linear algebra and analysis. There are two main classes of operator algebras:  $C^*$ -algebras and von Neumann algebras. Whereas the former have a more topological flavour (and their theory is thus often addressed as non-commutative topology), the latter has more measure theoretic and probabilistic sides and gives rise to non-commutative measure theory and non-commutative probability theory.

Von Neumann algebras themselves are already very intriguing, but the theory becomes even more interesting if one tries to understand subfactors, i.e., the question how one von Neumann algebra can be embedded into another one. Vaughan Jones addressed this question in the 1980's and found an amazing link to knot theory. In the end this resulted in a new invariant for knots, the Jones polynomial, and earned Jones the Fields Medal. Whereas the first investigations of the subfactor problem were quite analytical, Jones introduced, motivated by the relation with knots, in the 1990's a more combinatorial and diagrammatical description, which goes under the name of "planar algebras". This can on one side be seen as a special example of the more general theory of "operads", but has on the other side also a very planar, i.e., non-crossing structure, which makes it resemble the combinatorics of free probability. There has been some interesting consequences coming out of this apparent connection, but the final word on this has not yet been spoken.

In the lecture, I will try to give an introduction into this circle of ideas and I will hopefully also convey some of the excitement of the subject. Formally, the lecture is a continuation of "Funktionalanalysis 2" by M. Weber from the Winter term 15/16. I will recall (but not prove) all the needed definitions and facts on von Neumann algebras and then present in detail the analytical and diagrammatic theory of subfactors and planar algebras. So it would be good to have some prior knowledge on operator algebras, and perhaps von Neumann algebras, but no prerequisites on subfactors or planar algebras are assumed.

**Zeit und Ort: Montags und donnerstags, jeweils 12 – 14 Uhr, im Seminarraum 6, Geb. E2 4**

Zu dieser Vorlesung wird eine Übung angeboten werden, so dass ein Schein mit 9 Leistungspunkten erworben werden kann.

Fragen zur Vorlesung können gerne an Tobias Mai (Zimmer 225 oder per Mail an [mai@math.uni-sb.de](mailto:mai@math.uni-sb.de)) gerichtet werden. Siehe auch:

<http://www.math.uni-sb.de/ag/speicher/lehre.html>