Definition 1 (Classical point of view): A finite dimensional **representation** of a group G is a grouphomomorphism $G \to GL_n(\mathbb{K})$ for some field $\mathbb{K}$.

Definition 2: $T : \mathbb{K}^n \to \mathbb{K}^m$ such that for two representations ρ, σ we have $\rho(g)T = T\sigma(g) \, \forall \, g \in G$ is called **intertwiner**.

Definition 3 (Categorical point of view): A representation of a group G (as one-object category) is a functor $G \to Vect_{\mathbb{K}}$. Intertwiners are then natural transformations between two such functors.

Definition 4 (C^*-category [NT13, Web17]): A category $\mathcal{C}$ is called C^*-category provided that:

(i) $\forall \, U, V \in Ob(\mathcal{C}) Mor(U, V)$ is a Banach space, composition of morphisms is bilinear and $\|ST\| \leq \|S\| \|T\|$

(ii) we have an antilinear contravariant functor "involution" $* : \mathcal{C} \to \mathcal{C}$: $* = \mathrm{id}$ on objects, and on morphisms $T \in Mor(U, V)$:

   (1) $T^{**} = T$,

   (2) $\|T^*T\| = \|T\|^2$, and $End(U)$ is a unital C^*-algebra $\forall \, U \in Ob(\mathcal{C})$,

   (3) $T^*T \in End(U) = Mor(U, U)$ is positive.

Definition 5 (C^*-tensor category [NT13]): A C^*-category is called C^*-tensor category provided that it is a tensor category and if furthermore:

(i) The associativity morphisms are **unitary**: $uu^* = \mathrm{id}, u^*u = \mathrm{id}$ and $L_1, R_1 : X \to X \otimes 1, 1 \otimes X$ are unitary.

(ii) $(S \otimes T)^* = S^* \otimes T^*$ for all morphisms S, T.

Definition 6 ((Semi-)Simplicity [NT13]): An object in a C^*-tensor category $U \in \mathcal{C}$ is called **simple** if $End(U) \cong \mathbb{C}1$. $\mathcal{C}$ is called **semisimple** provided that every object can be decomposed into a direct sum of simple objects.

Definition 7 (Fiber functor [NT13]): A tensor functor $F : \mathcal{C} \to$ f.d.-Hilb is called **fiber functor** if it is faithful and exact.

Example 8 (canonical fiber functor [NT13]):

$$F(U \overset{S}{\to} V) = H_U \overset{S}{\to} H_V$$

Definition 9 (Compact quantum group [NT13]): A compact quantum group is a pair (A, Δ), where A is a unital C^*-algebra and $\Delta : A \to A \otimes A$ is a unital *-homomorphism, called comultiplication:

(i) $(\Delta \otimes \mathrm{id})\Delta = (\mathrm{id} \otimes \Delta)\Delta$ coassociativity,

(ii) $(A \otimes 1)\Delta(A), (1 \otimes A)\Delta(A)$ are dense in $A \otimes A$: cancellation property.

Definition 10 ([NT13]): A representation of a compact quantum group A is an invertible element $U \in M_n(A) \cong M_n(\mathbb{C}) \otimes A$. If U is unitary, then we call the representation unitary.

Definition 11: A **Hopf *-algebra** is an involutive algebra A together with algebra *-homomorphisms $\Delta\colon A \to A \otimes A$, $\varepsilon\colon A \to \mathbb{C}$, $S\colon A \to A^{\mathrm{opp}}$, such that the following conditions hold:

$$(\mathrm{id} \otimes \Delta) \circ \Delta = (\Delta \otimes \mathrm{id}) \circ \Delta$$
$$(\mathrm{id} \otimes \varepsilon) \circ \Delta = \mathrm{id} = (\varepsilon \otimes \mathrm{id}) \circ \Delta$$
$$m \circ (S \otimes \mathrm{id}) \circ \Delta = \eta \circ \varepsilon = m \circ (\mathrm{id} \otimes S) \circ \Delta$$

Definition 12 ([NT13, Web17]): We denote by $\mathbb{C}[G]$:= the linear span of matrix coefficients of all f.d. unitary representations of G = algebra generated by these.

Theorem 1 ([Web17, NT13]): *For any CQG $\mathbb{C}[G], \Delta$ is a Hopf-*-algebra. Conversely, any Hopf-*-algebra generated by matrix coefficients of f.d. unitary corepresentations is of that form for some CQG.*

Theorem 2 (Woronowicz's Tannaka Krein duality [NT13]): *Let $\mathcal{C}$ be a rigid C^*-tensor category, $F : \mathcal{C} \to$ f.d.- Hilb be a unitary fiber functor. Then there exist*

- *a compact quantum group G and*
- *a unitary monoidal equivalence $E : \mathcal{C} \to Rep(G)$*
- *such that F is naturally unitarily monoidally isomorphic to the composition of the canonical fiber functor with E.*
- *Furthermore, the Hopf *-algebra $(\mathbb{C}[G], \Delta)$ for such a G is uniquely determined up to isomorphism.*

Bibliography

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