ISem24 ⋊⋉ *C^*-algebras and dynamics*

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In the 1940s, Gelfand and Naimark introduced C^*-algebras, mainly in order to study representations of groups. It quickly developed into a research area on its own linking techniques from functional analysis and algebra in a fascinating way. Technically speaking, C^*-algebras are Banach algebras which are equiped with an involution satifying a particular norm condition

$$\|x^*x\| = \|x\|^2.$$

This condition forces a behaviour similar to the supremum norm on the algebra $C(X)$ of continuous, complex-valued functions on a compact Hausdorff space X. Indeed, such algebras are prototypes of commutative C^*-algebras and it is a fundamental theorem by Gelfand and Naimark that the converse is also true: For any commutative C^*-algebra there exists a compact space X such that the C^*-algebra is isomorphic to $C(X)$. This is the famous Gelfand duality between compact topological spaces and commutative C^*-algebras – turning the theory of (possibly noncommutative) C^*-algebras into a kind of noncommutative topology.
Noncommutative C^*-algebras come into play as soon as we move to dynamical systems, i.e., to actions of compact groups G on compact spaces X. Such dynamics may be studied in terms of the (typically noncommutative) C^*-algebra $C(X) \rtimes G$ and we may employ tools from the theory of C^*-algebras.

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Building on some basic knowledge on operators on Hilbert spaces, we will introduce C^*-algebras, spectra of Banach algebras and prove Gelfand duality, which provides us with the powerful tool of functional calculus for continuous functions. We will then turn to unitizations, positive elements, approximate units, ideals, states and representations, eventually proving that all C^*-algebras may be represented concretely on a Hilbert space by means of the so-called GNS construction. In the last third of the lecture, we will study actions of groups on topological spaces, dynamical systems and crossed products of C^*-algebras.

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Lecture Phase (October 2020 — February 2021). Electronic lecture notes are provided weekly via the ISem24 website. The lectures are self-contained and they include some exercises. In local groups, ideally led by a local coordinator, students from all over the world read these notes and discuss them in an online chat room.

Project Phase (March 2021 — June 2021). In small international groups led by some of the coordinators, the participants work on various projects supplementing Phase 1.

Final Workshop (6—12 June 2021). A one-week workshop takes place at the Bundeshöhe in Wuppertal, Germany. The projects from Phase 2 are presented and there are talks by experts in the field of C^*-algebras.

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The *Internet Seminar* (ISem) has been organized every year since 1997 by several working groups from Austria, Germany, Hungary, Italy and the Netherlands. It has been founded by the functional analysis groups in Tübingen, Ulm and Karlsruhe. The ISem introduces Master's and PhD students to modern topics in functional analysis related to evolution equations. See below for a web link on the history of the ISem and its structure in three phases. With ISem24, we follow this structure.

ISem in general @ `http://www.math.kit.edu/iana3/seite/isem/en`
ISem24 @ `https://www.math.uni-sb.de/ag/speicher/ISem24.html`