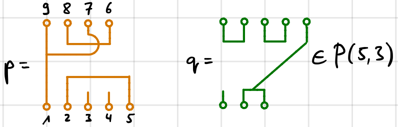
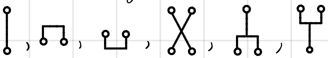
(set) partitions

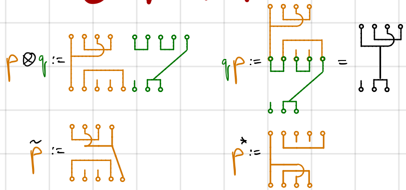


$P := (P(k,\ell))_{k,\ell \in \mathbb{N}_0}$

Ex:



Category of partitions



$\mathcal{C} = (\mathcal{C}(k,\ell))_{k,\ell\in\mathbb{N}_0}$ category of partitions

$:\Leftrightarrow$ $\mathcal{C}$ closed under above operations & $\in \mathcal{C}$

Tannaka-Krein / Schur-Weyl

non-orth. easy qu.groups?

(non-unitary)

$P_0 := \{ p \in P \mid p, p^* \in H_n(X) \Rightarrow , \in H_n(X) \}$

Prop.: $p \in P_0$, $p, p^* \in H_n(X) \Rightarrow (A_n(X), u) \subseteq O_n^+$ easy

Thm: $p \in P(0,\ell)$, $p \neq \dots$. $p \in P_0$ if

(a) ℓ odd, (b) ℓ even, $\nexists$ block of size 1, $\exists$ block of size ≥ 3 in p

(c), (d), ...

Q: $\in P_0$? $\in P_0$?

C*-algebraic relations

$p \in P(k,\ell)$ $\quad \forall \alpha_1, \dots \alpha_k, \beta_1, \dots \beta_\ell \in \{1, \dots, n\}$

$$R(p): \sum_{\gamma_1,\dots\gamma_k=1}^{n} \delta_p(\gamma,\beta)\, u_{\gamma_1\alpha_1} \dots u_{\gamma_k\alpha_k} = \sum_{\gamma_1',\dots\gamma_\ell'=1}^{n} \delta_p(\alpha,\gamma')\, u_{\beta_1\gamma_1'} \dots u_{\beta_\ell\gamma_\ell'}$$

& $u_{ij} = u_{ij}^*$ $\quad i,j \in \{1, \dots, n\}$

$R(\,)$: $u_{ij} = u_{ij}$ $\qquad$ $R(\,,\,)$: $uu^t = u^t u = 1$

$R(\,)$: $u_{ij} u_{k\ell} = u_{k\ell} u_{ij}$ $\qquad$ $R(\,,\,)$: $u_{mi} u_{mj} = \delta_{ij} u_{mi}$

& $u_{im} u_{jm} = \delta_{ij} u_{im}$

$A_n(X) := C^*(1, u_{ij}, i,j \in \{1,..,n\} \mid u_{ij} = u_{ij}^* \;\&\; R(p)\; \forall p \in X)$

"algebraic approach"

$H_n(X) := \{ p \in P \mid R(p) \text{ hold in } A_n(X) \} \supseteq X$

Thm: a) $p, q \in H_n(X) \Longrightarrow p \otimes q, qp, \tilde{p} \in H_n(X)$

b) $,\ \in H_n(X)$. $\quad p \in H_n(X) \Rightarrow p^* \in H_n(X)$

Cor.: $\mathcal{C} = \langle p_1, \dots, p_m \rangle \Rightarrow A_n(\mathcal{C}) = A_n(\{p_1, \dots, p_m, , \})$

Thm: $p \in P(0,\ell)$, $p \neq \dots$, $p, p^* \in H_n(X) \Rightarrow (A_n(X), u)$ CMQG

(easy) quantum group

$G = (A, u)$ Compact matrix quantum group $:\Leftrightarrow$ [Woronowicz 80's]

- A unital C^*-algebra generated by u_{ij}, $i,j = 1, \dots, n$
- $u = (u_{ij})$, $\bar{u} = (u_{ij}^*)$ are invertible
- $\Delta : A \to A \otimes_{\min} A$, $u_{ij} \mapsto \sum_k u_{ik} \otimes u_{kj}$ *-hom.

G "easy quantum group" $:\Leftrightarrow$ [Banica, Speicher 2009] $A = A_n(\mathcal{C})$, $\mathcal{C}$ categ. of part

C*-algebras and quantum spaces arising from partitions **Moritz Weber, Saarland U, Germany**

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