

NONCOMMUTATIVE ERGODIC THEOREMS

MORITZ WEBER (SAARBRÜCKEN)

The ergodic theorems presented in the internetseminar are certain statements about convergence in measurable or topological spaces. Since about 80 years, also noncommutative measurable and topological spaces are being studied: namely von Neumann algebras and C^* -algebras [B06]. They form the basis for many other concepts in mathematics involving noncommutativity such as quantum physics, (compact) quantum groups and Connes' noncommutative geometry, with links to quantum information theory, just to name a few.

In this project we will first make ourselves acquainted with the basic interpretation of von Neumann algebras resp. C^* -algebras as noncommutative measurable resp. topological spaces. This is based on Gelfand-Naimark's Theorem [B06, II.2.2.4, II.2.2.6]. We will learn how statements about spaces and measures are generally transformed into statements on von Neumann algebras and states.

We will then turn to noncommutative versions of some of the ergodic theorems presented in the internetseminar. Our focus is on Lance's [L76] article from 1976 and possibly other results mentioned therein. Upon the interests of the participants of this project, we may also take into account more modern articles and approaches such as the one by Junge and Xu [JX07], those by Huang [HW17, H16a, H16b], for instance his Quantum Szemerédi Theorem, or the Rokhlin dimension/property in the theory of C^* -algebras [HWZ15] and its link to dynamical systems of C^* -algebras. However, this is optional.

Prerequisites: Ideally, the participants of this project have some basic knowledge on C^* - and von Neumann algebras. However, this may also be gained while working on the project. In case, some participant has some more advanced interests in von Neumann algebras, compact quantum groups or dynamical systems of C^* -algebras, the above mentioned optional additional aspects may be taken into account individually.

References

- [B06] Blackadar, B. *Operator algebras*. Encyclopaedia of Mathematical Sciences **122**, Theory of C^* -algebras and von Neumann algebras, Operator Algebras and Non-commutative Geometry, III, Springer-Verlag, Berlin, 2006.
- [HWZ15] Hirshberg, Ilan and Winter, Wilhelm and Zacharias, Joachim. Rokhlin dimension and C^* -dynamics. *Comm. Math. Phys.* **335** (2), 637–670, 2015.
- [HW17] Huang, Huichi and Wu, Jianchao. Ergodic invariant states and irreducible representations of crossed product C^* -algebras, *J. Operator Theory* **78** (1), 159–172, 2017.
- [H16a] Huang, Huichi. Mean ergodic theorem for amenable discrete quantum groups and a Wiener-type theorem for compact metrizable groups, *Anal. PDE* **9** (4), 893–906, 2016.
- [H16b] Huang, Huichi. The quantum group fixing a sequence of finite subsets, *arXiv:1607.07276*, 2016.
- [JX07] Junge, Marius and Xu, Quanhua. Noncommutative maximal ergodic theorems *J. Amer. Math. Soc.* **20** (2), 385–439, 2007.
- [L76] Lance, E. Christopher. Ergodic theorems for convex sets and operator algebras *Invent. Math.* **37** (3), 201–214, 1976.