Universität des Saarlandes
FR 6.1 Mathematik
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5. Exercises for Algebraic Numbertheory

Exercise 1.

Consider the numberfields $K_\pm = \mathbb{Q}(\sqrt{\pm 2})$ and their rings of integers $\mathcal{O}_\pm$.
Write down their Minkowski-spaces $K_{\pm,\mathbb{C}}$ and $K_{\pm,\mathbb{R}}$ and draw the lattice $\mathcal{O}_\pm \subset K_{\pm,\mathbb{R}}$ in a suitable way.
What is the volume of each of these lattices?

Exercise 2.

Let $p \in \mathbb{N}$ be an odd prime number and ζ a primitive p-th root of unity.
Show the equality $\mathbb{Z}[\zeta]^\times = (\zeta)\mathbb{Z}[\zeta + \zeta^{-1}]^\times$ of groups of units.
Show furthermore, that $\mathbb{Z}[\zeta]^\times = \{\pm\zeta^m(1+\zeta)^n \mid 0 \leq m < 5, n \in \mathbb{Z}\}$ if and only if $p = 5$.
Hint: You can use, that $\mathbb{Z}[\zeta]$ *is the ring of integers of* $\mathbb{Q}(\zeta)$.

The exercises will be discussed in the question session at Monday the 04. 12. 2017 in SR9 at 16 (ct) before the presence exercises.