Universität des Saarlandes
FR 6.1 Mathematik
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1. Exercise sheet for Geometric Group Theory

Exercise 1.

Let Γ be a connected graph. We call an edge $e \in E(\Gamma)$ a *separating edge* if there exist two vertices $A, B \in V(\Gamma)$ such that every edge-path from A to B passes over e (that means e separates A from B).

a) Show that an edge $e \in E(\Gamma)$ is not separating if and only if it is contained in a simple cycle. A simple cycle is a cycle which passes over each vertex at most once, that means each vertex appears at most once as an origin vertex in the corresponding edge-path.

b) Let ϕ be a graph-automorphism of Γ. Show that ϕ preserves separating edges, that is $e \in E(\Gamma)$ is a separating edge if and only if $\phi(e)$ is also a separating edge.

c) Let $\Gamma(G, S)$ be a Cayley-graph with a separating edge. Show that G has infinite order.

Exercise 2.

Let Γ, Γ' be two graphs and $\phi : \Gamma \to \Gamma'$ be a graph-morphism. Show that ϕ is a graph-isomorphism if and only if ϕ is bijective on the edges and vertices.

Exercise 3. $(*)$

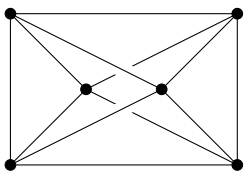
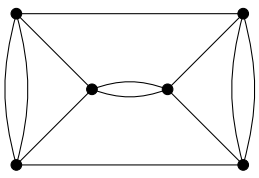
A tree is a connected graph without cycles. Show that any automorphism of a finite tree either fixes a vertex or „flips“ an edge, that means it sends e to $\overline{e}$.

Exercise 4.

Let G be a finitely generated group. Show that there exists a connected graph Γ such that G is isomorphic to the automorphism group of Γ.
Hint: The Cayley-graph is a good start, but its automorphism group might be to big. Maybe there is a way to eliminate some of the automorphisms?

Exercise 5.

We consider the two following graphs

$\Gamma :=$  and $H :=$ 

a) How many elements do the automorphism groups of Γ and H have?

You can hand in the exercise sheet until Monday the 25. 04. 2022 at 2 pm. Either give it directly to Christian Steinhart or throw it into box 47 in the basement in building E 2.5. The exercises marked with a $(*)$ might be a little bit trickier.

b) One of the two graphs is a Cayley-graph. Which one?
For the keen: What is the corresponding group and generators of the Cayley-graph?

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