Announcement and program for the seminar Dynamics in Algebra and Geometry

A dynamical system is a system consisting of a function that describes the time dependence of a point in a space. We can consider discrete time dependence as in population models of animals in a certain habitat, where we want to describe the number of animals at a fixed date every year. Or we can consider continuous time, for example if we want to describe the movement of a particle in space as in the Lorentz gas model. Sometimes it is useful to transfer a continuous dynamical system into a discrete one and study it with combinatorial tools. One example of this concept is provided by interval exchange maps, which are used to study flows on flat surfaces.
These examples already give an idea of the broadness of the research area, which plays an important role in disciplines as engineering, natural sciences and mathematical physics. The mathematical concepts which are used to describe dynamical systems include the theory of differential equations, ergodic theory, Lie groups, algebraic topology and even algebraic geometry. Among the mathematicians working on dynamical systems are for example John Milnor, Maxim Kontsevich or the first female fields medalist Maryam Mirzakhani.
In this seminar we will first concentrate on discrete dynamical systems and will hereby get to know iterated monodromy groups or Julia and Fatou sets. In the second half we want to focus on concepts of dynamical systems with continuous time range and their applications to K3 surfaces. Especially we want to discuss Oseledet's multiplicative ergodic theorem.

Prerequisites: This seminar adresses to master students who have fundamental knowledge in group theory and algebraic topology and who enjoy working autonomously on a topic.

Contact: If you have questions please contact **kany[at]math.uni-sb.de**.

Talk 1: Introduction to algebraic topology, monodromy of fibrations and coverings.

Speaker: Manuel Kany **Date, place: November 7, SR 9**

Goal of this talk is to explain the fundamental group functor [12, Theorem 2.3.7]. The functor is a generalization of the well known concept of monodromy of covering spaces.

- Provide important concepts from algebraic topology as far as they are useful for your talk. Chapter 1 in [12] could be useful.

- Define the homotopy lifting property [12, chapter 2 section 2].
- Explain [12, Theorem 2.3.7], compare section 5 in [10]
- Explain monodromy of covering spaces from this new point of view.

Literature: Cutler [2], Spanier [12], Palmer, Tillman [10]

Talk 2: Iterated monodromy groups.

Speaker: Leon Pernak **Date, place: November 14, SR 9**

Give definitions from algebraic perspective and dynamics perspective as in [8] or [7]. Discuss easier interesting properties and examples:

- Start with a short discussion of rooted trees: [7, section 2]
- Define iterated monodromy groups as in [7, section 3.1] or [8, section 2] and give examples.
- Explain Proposition 2.1 in [8] or Theorem 3.5 in [7].
- Continue with applications in holomorphic dynamics as in [7, section 4] or continue as in [8]. For example you can try to explain Hubbard's Twisted Rabbit Problem (see [1]).

Literature: Nekrashevych [8], Godillon [7]

Talk 3: Complex dynamics I: The Julia set and the Fatou set.

Speaker: Stevell Muller **Date, place: November 21, SR 9**

Define the Julia Set and the Fatou Set as in [8] or [9]. Give different examples for complex dynamics on Riemann surfaces as in section 3,4,5 of [9]:

- Repeat some basic facts about Riemann surfaces as far as you need them in your talk. You can hereby use for example [9, section 1,2].
- Define the Julia set and the Fatou set and discuss easier properties and examples on the Riemann Sphere as in [9, section 3].
- Present some interesting results on other Riemann surfaces [9, section 4].
- Finally give some examples [9, section 5]

Literature: Nekrashevych [8] and Milnor [9]

Talk 4: Complex Dynamics II: Local fixed point theory.

Speaker: Pascal Kattler **Date, place: November 28, SR 9**

Present some nice results and examples from the local fixed point theory. If you want to use [9, section 6,7], you can proceed as follows:

- Start with Koenigs Linearization Theorem [9, Theorem 6.1]. Proof uniqueness and existence.
- Present some corollarys from Koenigs Linearization Theorem.
- Present [9, Theorem 6.6] or [9, Theorem 6.7].
- Explain the Lea-Fatou Flower Theorem [9, Theorem 7.2].

Literature: Milnor [9]

Talk 5: Complex Dynamics III: Global fixed point theory.

Speaker: tba **Date, place: December 5, SR 9**

Present some nice results and examples from the global fixed point theory [9, section 9,10,11]. Start with section 9 in [9]. After that you can either focus on one of the sections 10, 11 in Milnor's notes or give an overview of both:

- Proof [9, Theorem 10.1], a result of Fatou.
- Proof [9, Theorem 11.1], a characterization of the Julia set. This theorem goes back to Julia and Fatou.

Literature: Milnor [9]

Talk 6: Introduction to Ergodic Theory.

Speaker: Konstantin Bogdanov **Date, place: tba**

Goal of this talk is to provide all the concepts from ergodic theory and measure preserving systems needed in the following talks. Especially:

- Measure preserving transformations [3, 2.1] and Ergodicity [3, 2.3].
- Group-invariant measures and ergodic measures. See for example [13, 2.1].

- Birkhoff's pointwise ergodic theorem [3, 2.6.4].

Literature: Einsiedler, Ward [3], Zimmer [13]

Talk 7: Amenability and cocycles.

Speaker: tba **Date, place: tba**

Definition and examples for amenable groups as in [13, 4.1]. Definition of cocycles as in [13, 4.2]. If there is time left, extend the notion of amenability to amenable actions with the help of cocycles:

- Define amenable groups, give examples and proof easier properties [13, 4.1]. You might have to explain Lie groups.
- Define cocycles, give examples and explain easy results fromy [13, 4.2]. This definition will be used and extended to vector bundles in one of the later talks.
- At the end you could extend the notion of amenable groups to amenable actions with the help of cocycles [13, 4.3].

Literature: Zimmer [13]

Talk 8: Oseledets Multiplicative Ergodic Theorem.

Speaker: tba **Date, place: tba**

Goal of this talk is to present chapter two of [4] as good as possible. And to maybe give one or two motivating examples for Oseledets multiplicative theorem and Lyapunov exponents:

- Define cocycles on vector bundles.
- Explain Oseledets multiplicative ergodic theorem and sketch a proof.
- Find nice examples, where Oseledet's theorem was used. For example in papers of mathematical physics as in the work `https://arxiv.org/abs/hep-th/9701164` of Kontsevich, or `https://arxiv.org/abs/1107.1810` of Delecroix, Hubert and Lelievre. Or easier ones :D.

Literature: Filip [4]

Talk 9: Introduction to K3 surfaces.

Speaker: tba **Date, place: tba**

Introduction talk to K3 surfaces.
Literature: tba

Talk 10, 11: Examples of appplications of dynamics on K3 surfaces.

Speaker: tba **Date, place: tba**

In this talks (maybe two are necessary) we want to see applications of dynamics on K3 surfaces. The speaker is free to give talks about whatever topic she or he likes. It would be nice to compare topics from Teichmüller dynamics and dynamics on K3 surfaces. Examples:

- Lyapunov exponents of families of K3 surfaces in the style of Kontsevich (after him many others) [6].
- Comparing Teichmüller dynamics with dynamics on K3 surfaces. See for example the survey [5].

Literature: Filip [5], [6], or papers of McMullen, or anybody else

References

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[7] S. Godillon, *Introduction to Iterated Monodromy Groups*, http://www.numdam.org/item/10.5802/afst.1362.pdf.

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[9] J. Milnor, *Dynamics in one complex variable, Introductory Lectures*, https://arxiv.org/pdf/math/9201272.pdf.

[10] M. Palmer, U. Tillman, *Configuration-mapping spaces and homology stability* (2021), Research in the Mathematical Sciences vol. 8(3), https://doi.org/10.1007%2Fs40687-021-00270-7.

[11] P. Schapira, *Algebra and Topology* (2007), https://webusers.imj-prg.fr/~pierre.schapira/LectNotes/AlTo.pdf.

[12] E. H. Spanier, *Algebraic Topology* (1989) Springer.

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