Weihnachtsworkshop 2022

Yael Algom-Kfir: Out(F_n) past and future

Abstract: We give a history of the theory of Out(F) the outer automorphism group of the free group – starting with Nielsen and Whitehead through the definition of Outer Space, and the free factor complex. We give some results in current research where we allow for the free group to be of infinite rank i.e. results on " Big Out".

Luca Battistella: Alternative compactifications of the moduli space of curves of genus two

Abstract: The moduli space of Riemann surfaces with n labeled points admits a celebrated compactification parametrising at worst nodal curves. This compactification, due to Deligne and Mumford, is (orbifold) smooth with normal crossing boundary and a projective coarse moduli space. Many questions on the birational geometry of the latter remain open. We take the point of view that these can be investigated by constructing new birational models by varying the stability condition. The Gorenstein condition identifies a class of singularities behaving similarly to nodes and similarly helpful for the purpose of enumerative geometry. I will survey some recent developments focussing on the case of genus two curves.

Etienne Bonnafoux: Polynomial bound on the rate of mixing of the earthquake flow

Abstract: The earthquake flow is a tool in hyperbolic geometry helping in various contexts from the Nielsen realization problem to the asymptotic counting of simple closed geodesics. Its ergodic properties remain as yet partially understood. We know that it is mixing with respect to the so-called Mirzakhani measure. But its mixing rate is unknown. In this presentation, I will explain how to bound the rate of mixing of the earthquake flow, it could at most be polynomial with a degree dependent on the topology of the surface.

Andrei Bud: Brill-Noether loci and strata of differentials

Abstract: When studying the birational geometry of the space $\mathcal{M}_g$ parametrizing genus g Riemann surfaces, Brill-Noether theory plays a fundamental role. We look at the Brill-Noether properties for a generic element in a stratum of differentials. Drawing parallels with the case of the moduli space $\mathcal{M}_g$, we study the birational geometry of strata of differentials. We present some results in this direction.

Ernst-Ulrich Gekeler: Non-Archimedian contour integration and applications

Abstract: In the first half of the talk, I will present some simple techniques from non-archimedean analysis and geometry, viz, the formalism of contour integration and residue calculus essentially due to GERRITZEN and VAN DER PUT. These are known since about 40 years, but unfortunately not really well-known. I want to publicize these techniques and propagate their use by presenting a non-trivial application to characteristic p arithmetic ($p > 0$). This will be the topic of the second half of the talk, where I deal, among other things, with power sums of polynomials over finite fields.

Martin Günther: Spaces with local chronological structure and the cosheaf of fundamental semicategories

Abstract: We examine topological spaces with a set of distinguished "timelike" paths satisfying some axioms inspired by lorentzian spacetimes. The timelike homotopy classes of paths in such a space give rise to a semicategory with interesting algebraic and topological properties. For example, we show that the isomorphism class of this "fundamental" semicategory uniquely determines the topology and isomorphism class of the original space. The van-Kampen theorem can also be generalized from groupoids to fundamental semicategories, which means that they actually form a semicategory-valued cosheaf.

Luke Jeffreys: Meanders, hyperelliptic pillowcase covers, and the Johnson filtration

Abstract: Pillowcase covers are branched covers of the four-times punctured sphere – the pillowcase – and arise naturally in a number of settings in low-dimensional topology. $[1,1]$-pillowcase covers are a special class of pillowcase cover with natural connections to filling pairs on surfaces and the theory of pseudo-Anosov diffeomorphisms. Indeed, minimal constructions of $[1,1]$-pillowcase covers give rise to constructions of so-called 'ratio-optimising' pseudo-Anosovs. In this talk, we will be interested in minimal constructions of $[1,1]$-pillowcase covers corresponding to filling pairs where both curves in the pair are separating curves on the surface. Such $[1,1]$-pillowcase covers allow us to construct ratio-optimising pseudo-Anosovs lying arbitrarily deep in the Johnson filtration – an important sequence of subgroups of the mapping class group of the surface. I will present the minimal constructions of these pillowcase covers in the hyperelliptic case, and discuss how these constructions depend crucially on combinatorial objects called meanders that have been well studied in mathematics, computer science and physics.

Anna Lenzhen: Teichmüller disks with small limit sets in Thurston boundary.

Abstract: We look at the sets of accumulation points of Teichmüller disks in the Thurston boundary of the Teichmüller space. It is well known from work of H. Masur, S. Kerckhoff and J. Smillie that almost every Teichmüller geodesic ray converges to a point in PMF. We show that unlike geodesic rays, Teichmüller disks with smallest possible limit sets are extremely rare.

Maria Beatrice Pozzetti: What are higher rank Teichmüller theories?

Abstract: Higher rank Teichmüller theories are connected components of character varieties of fundamental groups of surfaces only consisting of injective homomorphisms with discrete image. After explaining the parallel with Teichmüller theory, I will discuss some geometric properties of these representations that parallel properties of holonomies of hyperbolizations. Joint work with Jonas Beyrer.

Slade Sanderson: Constructing lattice surfaces with given Veech groups

Abstract: A translation surface is a collection of polygons together with identifications of pairs of edges via translation, and its Veech group is the group of Jacobians of orientation-preserving affine automorphisms of the surface. A classification of which groups are realized as Veech groups is unknown. We give a gentle, geometric introduction to translation surfaces and their Veech groups and present an algorithm which constructs all translation surfaces with a given (lattice) Veech group.

Jan-Christoph Schlage-Puchta: You have no choice –
The best set theory for Algebra and Geometry

In a naïve approach to set theory, every collection of objectss is a set. Once the need for an axiomatization becomes clear, most mathematicians settle for ZFC, that is, the essential properties of sets plus the axiom of choice.
I will first show that this choice is less natural than on emay think. I will then describe Solovay's set theory, where dependent choice holds, and every subset of the reals is measurable. It turns out that this set theory is particularly well suited to treat problems in algebra and geometry. I will show that in Solvay's universe several problems in group theory become easy, or have a different answer from ZFC, or become solvable at all. I will finally muse on the meaning of independence results to "ordinary" mathematics. tba

Christian Steinhart: Embeddings of Outer Space

Abstract: We will have a brief introduction of Outer Space and its Lipschitz metric. As we have seen in "Explosions in Outer Space" the isometry group of Outer Space comes from the $\mathrm{Out}(F_n)$-action on it. In particular isometries preserve the simplicial structure of Outer Space. We will see an example, that this might not be true if we consider isometric embeddings from Outer Spaces of different rank. Furthermore we will discuss two families of isometric embeddings coming from finite index subgroups and free factors. While isometric embeddings coming from free factors and finite index subgroups of F_2 can be continuously deformed into other isometric embeddings, it turns out that isometric embeddings induced by finite index subgroups from F_n for $n > 2$ exhibit some sort of rigidity.

Alexander Thomas: Dessins d'enfant and symmetry

Abstract: "A Dessin d'enfant, or children's drawing, is a bipartite graph embedded into a surface. Surprisingly, this data encodes an algebraic structure and a meromorphic function, the Belyi map. I present two special classes of dessins d'enfant with many symmetries. The first class leads to generalized Platonic solids, which are regular triangulations of surfaces, and to normal subgroups of the modular group PSL(2,Z). The second leads to statistical models (Ising model and percolation) on self-similar graphs and renormalization.

Moritz Weber: Quantum Permutations and Quantum Symmetries

Abstract: In the past decades a kind of ,,quantum mathematics" has evolved as a more and more coherent theory. It contains, amongst others, C*-algebras (aka noncommutative topology), von Neumann algebras (aka noncommutative measure theory), Connes's noncommutative (differential) geometry, Voiculescu's free probability theory and many more. In this mostly analytic setting, Woronowicz's quantum groups provide a suitable notion of quantum symmetry. In this talk, we will give a pedestrian approach to quantum symmetries: We will introduce quantum permutations purely in the language of linear algebra and sketch its use in graph theory (see for instance an exciting extension of Lovasz' homomorphism counts theorem from the 1960s). On the way, we will briefly mention the broader context of quantum mathematics, quantum groups and some links to quantum information theory. However, the talk will mostly use algebraic and combinatorial language only.